Distance-preserving graph contractions

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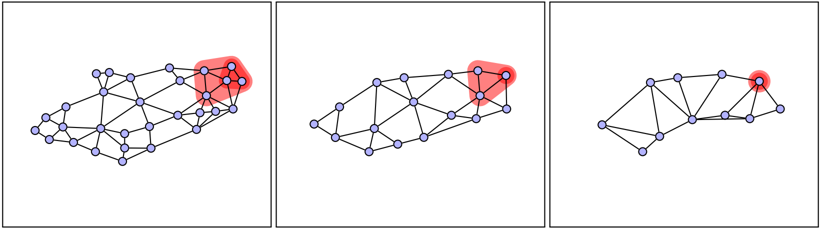
June 5, 2018



Distance-preserving graph compressions

Given: A graph $G = (V, E)$.

Goal: Find a 'small' graph that preserves (some) shortest-path distances (approximately).

Motivation: Less memory requirements/faster algorithm run times.

Examples: Spanners, Distance Preservers, Distance-Preserving Minors, . . .

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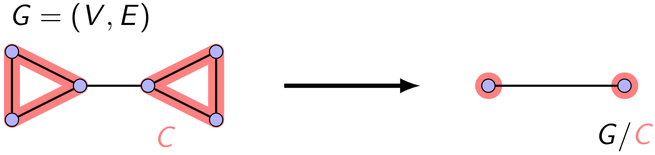
... and now ...

(α, β)-contraction

Given: An undirected and weighted graph $G = (V, E)$.

Operation: Edge contractions:



Constraint: Given $\alpha \geq 1, \beta \geq 0$:

$$\text{dist}_{G/C}(u, v) \geq \text{dist}_G(u, v)/\alpha - \beta \text{ for all } u, v \text{ in } V.$$

Goal: Find an (α, β)-contraction maximizing the number of deleted edges.

Results

<table>
<tr><th></th><th colspan="4">Graph classes</th></tr>
<tr><th></th><th>Path</th><th>Tree</th><th>Cycle</th><th>General</th></tr>
<tr><td>addit. (α=1), unit lg.</td><td rowspan="2">$\mathcal{O}(n)$</td><td>$\mathcal{O}(n)$</td><td rowspan="2">$\mathcal{O}(n)$</td><td rowspan="2">$m^{\frac{1}{2}-\varepsilon}$-inapx.</td></tr>
<tr><td>affine (α, β), unit lg.</td><td></td></tr>
<tr><td>addit. (α=1)</td><td colspan="2" rowspan="2">$\mathcal{O}(n^3)$</td><td rowspan="2">NP-hard</td><td rowspan="2">$n^{1-\varepsilon}$-inapx.</td></tr>
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</table>

Algorithms and hardness

Asymptotic bounds for unit length edges

	# of edges in G/C	Time
$(\alpha, \beta) = (2k-1, 1)$	$n^{1+1/k}$	$\mathcal{O}(m)$
$(\alpha, \beta) = (2\log_2 n - 1, 1)$	$2n$	$\mathcal{O}(m)$
$(\alpha, \beta) = (k-1, 1)$	$\Omega(n^{1+1/k})$	—
$(\alpha, \beta) = (1, k)$	$m - km/(2n)$	$\mathcal{O}(m)$
$(\alpha, \beta) = (1, k)$	$\mathcal{O}(n^2/k)^*$	$\mathcal{O}(m)$
$(\alpha, \beta) = (1, \mathcal{O}(1))$	$\Omega(n^{4/3-o(1)})^\dagger$	—
min. degree D	# of vertices in G/C	Time
$(\alpha, \beta) = (5, 1)$	n/D	$\mathcal{O}(m)$
$(\alpha, \beta) = (k, 1)$	$\Omega(n/(kD))$	—

* [Bernstein, Chechik '16]

† [Abboud, Bodwin '16]

Results

	Graph classes: Path	Tree	Cycle	General
addit. (α=1), unit lg.	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$m^{\frac{1}{2}-\varepsilon}$-inapx.
affine (α, β), unit lg.	$\mathcal{O}(n)$	$\mathcal{O}(n^3)$	$\mathcal{O}(n)$	$m^{\frac{1}{2}-\varepsilon}$-inapx.
addit. (α=1)	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$	NP-hard	$n^{1-\varepsilon}$-inapx.
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Multiplicative Spanners/Contractions with Unit Lengths

Distortion	Size	Notes
Spanner		
$(2k-1,0)$	$\mathcal{O}(n^{1+1/k})$	[Althöfer, Das, Dobkin, Joseph, Soares '93]
$(2k-1,0)$	$\Omega(n^{1+1/k})^*$	[Peleg, Schäffer '89]
Contraction		
$(2k-1,1)$	$n^{1+1/k}$	**new**
$(k-1,1)$	$\Omega(n^{1+1/k})^*$	**new**

*Depending on Erdős' girth conjecture.

Summary

Contractions are . . .

- . . . a new model to compress graphs while preserving distances.
- . . . conceptually similar to spanners with significant differences.
- . . . hard to compute exactly on most graphs.
- . . . promising with respect to asymptotic bounds.

Thank you!